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SOLUTIONS OF A PAIR OF CONSERVATION LAWS WITH NONCONVEX FLUX FUNCTION (2)

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Abstract. We study a pair of hyperbolic conservation laws which is derived from a nonlinear lattice and is a model for the motion in an elastic body. Adjacent material particles in the body are assumed to interact via the potential V expressed by a single term with an exponent $\gamma(\geq 4)$. Our model is similar to the isentropic Euler system but allows dependent variables to vary from negative to positive values. So the flux function is non-convex. It is proved that the Riemann problem is solved in the whole space. The solutions are continuous with respect to γ. The wave motion and the dissipation of mechanical energy are studied numerically for smooth initial data. Shocks waves decay slowerly with the increase of γ, namely for the stronger nonlinearity of interaction. However, the asymptotic decay rate of the total mechanical energy in shocks is almost little influenced by γ.
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1 Introduction

We study the motion in one-dimensional elastic body. The motion approximates that of a nonlinear lattice [6, 7]. In the body, adjacent material particles interact via a potential V. The potential is assumed to be a smooth function of the deformation gradient $U_x(x,t)$ at the point x and time t, $V(U_x(x,t))$. Then the equation of motion is

$$U_{tt} = (V'(U_x))_x. \tag{1}$$

It is assumed that $V''(U_x) > 0$ [3], and the impenetrability of matter restricts $U_x(x,t) > -1$.

Introducing the variables $\tilde{u} = U_x$ and $\tilde{v} = U_t$, we obtain,

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix}_t + f_x = 0, \qquad f = \begin{pmatrix} -\tilde{v} \\ (-V'(\tilde{u}))_x \end{pmatrix}. \tag{2}$$

The system (2) admits the conservation of mechanical energy,

$$E_t(\tilde{u},\tilde{v}) + F_x(\tilde{u},\tilde{v}) = 0. \tag{3}$$