

## ON EXISTENCE AND ITERATIVE ALGORITHMS OF SOLUTIONS FOR MIXED NONLINEAR VARIATIONAL-LIKE INEQUALITIES IN REFLEXIVE BANACH SPACES

Zeqing Liu<sup>1</sup>, Jeong Sheok Ume<sup>2</sup> and Shin Min Kang<sup>3</sup>

<sup>1</sup>Department of Mathematics, Liaoning Normal University,  
P. O. Box 200, Dalian, Liaoning, 116029, People's Republic of China

<sup>2</sup>Department of Applied Mathematics  
Changwon National University, Changwon 641-773, Korea

<sup>3</sup>Department of Mathematics and Research Institute of Natural Science  
Gyeongsang National University, Chinju 660-701, Korea

**Abstract.** In this paper, we introduce and study a new class of mixed nonlinear variational-like inequalities in reflexive Banach spaces. By applying the auxiliary principle technique due to Cohen, the minimax inequality due to Ding and Tan, and the fixed point theorems due to Banach and Kirk, respectively, we establish the existence and uniqueness theorems of solutions for the mixed nonlinear variational-like inequalities, suggest two general algorithms and prove the convergence of the iterative sequences generated by the algorithms. Our results extend, improve and unify the results due to Lions and Stampacchia, Noor, Bose, Zhang and Xiang, Ding and Tarafdar, Ding, Glowinski and others.

**Keywords.** Minimax inequality, Auxiliary principle technique, Mixed nonlinear variational-like inequality, Fixed point, Contraction mapping, Nonexpansive mapping,  $\eta$ -relaxed Lipschitz mapping,  $\eta$ -relaxed monotone mapping, Algorithm, Convergence, Reflexive Banach space.

**AMS (MOS) subject classification:** 47J20, 49J40.

## 1 Introduction

It is well known that the variational inequality theory provides us a unified, simple, elegant and natural framework to study a wide class of linear and nonlinear problems arising in optimization, economics, transportation, free boundary valued problems, etc, see [1]-[4], [6]-[9] and [11]-[22] and the references therein. In 1967, Lions and Stampacchia [11] established a few existence results of solutions for elliptic variational inequalities in Hilbert spaces. In 1987 Noor [16], [17] extended the results due to Lions and Stampacchia [11] and studied the existence and uniqueness of solutions for a class of variational inequalities involving a continuous bilinear form  $a(u, v)$ , a nonlinear form  $b(u, v)$  and an antimonotone mapping. In 1990, Bose [1] obtained some results similar to Noor [16], [17] under different conditions. In 2002, Liu, Ume and Kang [15] introduced and studied a kind of general strongly nonlinear