Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 99-122

EIGENVALUE PROBLEMS AND BIFURCATION OF NONHOMOGENEOUS SEMILINEAR ELLIPTIC EQUATIONS ON UNBOUNDED CYLINDER DOMAINS

Tsing-San Hsu

Center for General Education
Chang Gung University, Kwei-San, Tao-Yuan 333, Taiwan

Abstract. In this article, we consider the eigenvalue problems of Dirichlet problem

$$-\Delta u + u = \lambda(f(u) + h(x)) \text{ in } \Omega, u > 0 \text{ in } \Omega \text{ }, u \in H_0^1(\Omega), \qquad (*)_\lambda$$

where $\lambda > 0$, $N \geq 2$, and Ω is an unbounded cylinder domain in $\mathbb{R}^N$. Under some suitable conditions on f and h, we show that there exists a positive constant λ^* such that $(*)_\lambda$ has at least two solutions if $\lambda \in (0, \lambda^*)$, a unique positive solution if $\lambda = \lambda^*$ and no solution if $\lambda > \lambda^*$. We also obtain some bifurcation results of the solutions at $\lambda = \lambda^*$.
Keywords. eigenvalue problems, bifurcation, elliptic equation, unbounded cylinder.
AMS (MOS) subject classification: 35J20, 35J25, 35J60.

1 Introduction

Throughout this article, let $N = m + n \geq 2$, $n \geq 1$, $2^* = \frac{2N}{N-2}$ for $N \geq 3$, $2^* = \infty$ for $N = 2$, $x = (y, z)$ be the generic point of $\mathbb{R}^N$ with $y \in \mathbb{R}^m$, $z \in \mathbb{R}^n$, $N = m + n \geq 2$, $n \geq 1$ and Ω be an unbounded cylinder domain in $\mathbb{R}^N$.

Consider the following eigenvalue problems:

$$\begin{cases} -\Delta u + u = \lambda(f(u) + h(x)) \text{ in } \Omega, \\ u \in H_0^1(\Omega), \ u > 0 \text{ in } \Omega, \end{cases} \qquad (1.1)_\lambda$$

where $\lambda > 0$, $0 \in \omega \subseteq \mathbb{R}^m$ is a smooth bounded domain, $\Omega = \omega \times \mathbb{R}^n$ is an unbounded domain in $\mathbb{R}^N$, $h(x) \in L^2(\Omega) \cap L^{q_0}(\Omega)$ for some $q_0 > N/2$ if $N \geq 4$, $q_0 = 2$ if $N = 2, 3$, $h(x) \geq 0$, $h(x) \not\equiv 0$ and f satisfies the following conditions:

(f1) $f \in C^1([0, +\infty), \mathbb{R}^+)$, $f(0) = 0$ and $f(t) \equiv 0$ if $t < 0$;

(f2) there is a positive constant C, such that

$$|f(t)| \leq C\left(|t| + |t|^p\right), \text{ for some } 1 < p < 2^* - 1;$$

(f3) $\lim\limits_{t \to 0} t^{-1} f(t) = 0$;