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EXISTENCE OF OPTIMAL CONTROLS FOR DISTRIBUTED SYSTEMS INVOLVING DYNAMIC BOUNDARY CONDITIONS

S. Kerbal[1] and N.U. Ahmed[2]

[1]Department of Mathematics and Statistics
Sultan Qaboos University, Muscat, Oman
[2]Department of Mathematics and School of Information Technology and Engineering
University of Ottawa, Ottawa, Canada

Abstract. In this paper we use the calculus of variation, a classical technique to prove the existence of an optimal control for Lagrange type control problem subject to a semilinear systems governed by B-evolutions, that is for systems involving dynamics on the boundary. For motivation we present an example of heat transfer problem arising in the nuclear reactor.
Keywords. B-evolution systems, semi-linear systems, dynamic boundary control problem, semigroup, generating and closed pair of operators, optimal control , Lagrange problem.

1 Introduction

Many physical systems, with dynamic boundary conditions has applications in multi-phase problems in physics and engineering such as heat transfer and Navier-Stokes equation.

For motivation let us consider the following heat transfer equation with dynamic boundary condition. Let $\Omega \subset R^n, (n = 1, 2, 3)$ be an open bounded domain with smooth boundary which consists of two parts $\partial\Omega \equiv \Gamma_0 \cup \Gamma_1$. The material (e.g.fluid) in the interior of the domain receives heat energy through the boundary Γ_1 from an external source distributed on the exterior of the boundary layer Γ_1. Taking into account the dynamics of heat source the problem can be modelled as follows:

$$\begin{cases} (\dfrac{\partial}{\partial t})T(t,\xi) = div(k(\xi)\ \nabla T) + v.\nabla T + f(t,\xi,T(t,\xi)), t > 0, \xi \in \Omega \\ T(t,\xi)|_{\Gamma_0} = 0, \\ (\dfrac{\partial}{\partial t})(T(t,\xi)|_{\Gamma_1}) = -\beta D_\nu T(t,\xi)|_{\Gamma_1} + g(t,\xi,T(t,\xi)|_{\Gamma_1},u), \\ T(0,\xi) = T_0(\xi), \xi \in \Omega, T(0,\xi)|_{\Gamma_1} = T_1(\xi), \xi \in \Gamma_1. \end{cases} \quad (1)$$

Here T denotes the space-time temperature distribution in the interior of the