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TOPOLOGICAL SEQUENCE ENTROPY AND TOPOLOGICAL DYNAMICS OF INTERVAL MAPS

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Abstract. The relationship between the topological sequence entropy and the topological dynamics of a continuous interval map is studied. Some differencies between the classical topological entropy and the topological sequence entropy are found.
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1 Introduction and statement of results

Let (X, d) be a metric compact space and let $f : X \to X$ be a continuous self-map on X. The pair (X, f) is called a *dynamical system.* If $n \in \mathbb{N}$, then $f^n := f \circ f^{n-1}$, $f^1 := f$ and f^0 is the identity on X. For $x \in X$, the sequence $\{f^n(x) : n \in \mathbb{N}\}$ is called the *orbit* of x, denoted by $\mathrm{Orb}_f(x)$.

In what follows $A = \{a_i\}_{i=1}^{\infty}$ always denote an strictly increasing sequence of positive integers. We are going to introduce the notion of topological sequence entropy of f with respect to A (see [11]). Let $Y \subseteq X$ and fix $\varepsilon > 0$ and $n \in \mathbb{N}$. A subset $E \subset Y$ is said to be $(A, n, \varepsilon, Y, f)$–separated if for any $x, y \in E$, $x \neq y$, there is $i \in \{1, ..., n\}$ such that $d(f^{a_i}(x), f^{a_i}(y)) > \varepsilon$. Denote by $s_n(A, \varepsilon, Y, f)$ the cardinality of any maximal $(A, n, \varepsilon, Y, f)$–separated subset of Y. Define

$$s(A, \varepsilon, Y, f) := \limsup_{n\to\infty} \frac{1}{n} \log s_n(A, \varepsilon, Y, f),$$

and then, the topological entropy of f is

$$h_A(f) = \lim_{\varepsilon\to 0} s(A, \varepsilon, X, f).$$

If $A = \mathbb{N}$, then $h_A(f) = h(f)$ is the classical topological entropy of f introduced in [4]. Finally, define

$$h_\infty(f) := \sup\{h_A(f) : A \text{ is an increasing sequence of nonnegative integers}\}.$$

Topological sequence entropy is an useful tool to characterize interval and circle maps which are chaotic in the sense of Li and Yorke (see [10] and [12]).