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SOME EXISTENCE AND UNIQUENESS RESULTS FOR IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS WITH VARIABLE TIMES IN FRÉCHET SPACES

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Abstract. In this paper, a recent nonlinear alternative for contraction maps in Fréchet spaces due to Frigon and Granas is used to investigate the existence and uniqueness of solutions for impulsive functional differential equations in Féchet spaces.
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1 Introduction

This paper is concerned with the existence of solutions of initial value problems (IVP for short) for first order functional differential equations with impulsive effects of the form

$$y'(t) = f(t, y_t),\ a.e.\ t \in J = [0, \infty),\ t \neq \tau_k(y(t)),\quad k \in \mathbb{N}^*, \tag{1}$$

$$y(t^+) = I_k(y(t)),\quad t = \tau_k(y(t)),\ k \in \mathbb{N}^*, \tag{2}$$

$$y(t) = \phi(t) \quad t \in [-r, 0], \tag{3}$$

where $f : J \times D \to \mathbb{R}^n$ is a given function, $\mathbb{N}^* = \{1, 2, 3, \dots\}$, $\tau_k : \mathbb{R}^n \to \mathbb{R}$, $I_k \in C(\mathbb{R}^n, \mathbb{R}^n)$,

$$D = \{\psi : [-r, 0] \to \mathbb{R}^n \mid \psi \text{ is continuous everywhere except for a finite number of points } \bar{t} \text{ at which } \psi(\bar{t}) \text{ and } \psi(\bar{t}^+) \text{ exist and } \psi(\bar{t}^-) = \psi(\bar{t})\},$$

$\phi \in D$, and $0 < r < \infty$. For any function y defined on $[-r, \infty)$ and any $t \in J$, we let y_t denote the element of D defined by

$$y_t(\theta) = y(t + \theta),\quad \theta \in [-r, 0].$$