Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 13-26

INITIAL VALUE PROBLEMS IN INFINITE INTERVAL OF FIRST ORDER NONLINEAR IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. In this paper, by means of the monotone iterative technique and a comparison result, the existence of minimal and maximal solutions of an initial value problem in an infinite interval for first order impulsive integro-differential equations in Banach spaces is obtained. An example is given to demonstrate the application of our main results.
Keywords. impulsive integro-differential equation; infinite interval; monotone iterative technique; completely continuous operator; contraction mapping principle.
AMS (MOS) subject classification: 34B15, 34B25.

1 Introduction

In this paper, we consider the following initial value problem (IVP) for nonlinear first order impulsive integro-differential equations of Volterra type in a real Banach space $(E, ||\cdot||)$:

$$\begin{cases} x' = f(t, x, Tx), \forall t \in J, t \neq t_k, \\ \triangle x|_{t=t_k} = I_k(x(t_k)), k = 1, 2, \cdots, \\ x(t_0) = x_0, \end{cases} \tag{1.1}$$

where $f \in C[J \times E \times E, E]$, $J = [0, \infty)$, $I_k \in C[E, E]$ $(k = 1, 2, \cdots)$, $0 = t_0 < t_1 < \cdots < t_k < \cdots < \bar{t} < \infty$, $\lim\limits_{k\to\infty} t_k = \bar{t}$ and

$$(Tx)(t) = \int_0^t k(t, s)x(s)ds, \tag{1.2}$$

$k \in C[D, \mathbb{R}_+] \bigcap L^2(J)$, $D = \{(t, s) \in J \times J : t \geq s\}$, $\mathbb{R}_+ = [0, \infty)$, $\triangle x|_{t=t_k}$ denotes the jump of $x(t)$ at $t = t_k$, i.e., $\triangle x|_{t=t_k} = x(t_k^+) - x(t_k^-)$, here $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively.