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FROM POINT DISSIPATIVE TO COMPACT DISSIPATIVE - ADDENDUM TO "SOME COUNTEREXAMPLES IN DISSIPATIVE SYSTEMS"[1]

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Abstract. In this note certain notions of asymptotic smoothness and asymptotic compactness are considered, which are suitable to show that, for a continuous map T in a general metric space V, point dissipative implies compact dissipative. Different relations among these mentioned notions are investigated and some weak conditions that lead to compact dissipativeness of T are discussed. Additional remarks concerning α-contracting maps are also included.

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1 Introduction

It is known that, if T is a continuous map on a metric space V and J is a compact set, then J attracts neighborhoods of points if and only if J attracts neighborhoods of compact sets. The latter is equivalent to J attracting compact sets provided that T is asymptotically smooth. Stronger assumption of complete continuity of T is needed to generalize this equivalence and prove that bounded dissipative is equivalent to point dissipative.

In Cooperman [3] and in Cholewa and Hale [2] a number of examples are considered which either relates different notions of dissipativeness or show that they are unrelated. It is shown in particular that point dissipative and locally point dissipative are not equivalent for asymptotically smooth maps and that compact dissipative α-contracting maps may not be bounded dissipative. In particular, for the latter maps a maximal compact invariant

[1]J. W. Cholewa, J. K. Hale, Some counterexamples in dissipative systems, *Dynam. Contin. Discrete Impuls. Systems.* 7 (2000), 159-176.