Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 137-145

A GENERIC CONVERGENCE THEOREM FOR CONTINUOUS DESCENT METHODS IN BANACH SPACES

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Abstract. We study continuous descent methods for minimizing convex functions defined on general Banach spaces and prove that most of them (in the sense of Baire category) converge.

Keywords. Complete uniform space, convex function, descent method, generic property, initial value problem, minimization problem.

AMS (MOS) subject classification: 37L99, 47J35, 49M99, 54E35, 54E50, 54E52, 90C25.

1 Introduction

In this paper we continue our studies of descent methods. This is an important topic in optimization theory and in dynamical systems; see, for example, [1-3, 6-11]. Given a continuous convex function f on a Banach space X, we associate with f a complete metric space of vector fields $V : X \to X$ such that $f^0(x, Vx) \leq 0$ for all $x \in X$. Here $f^0(x, u)$ is the right-hand derivative of f at x in the direction of $u \in X$. In [2] we studied the convergence of the values of the function f to its minimum along the trajectories of continuous dynamical systems governed by such vector fields and established a convergence result for most of them. Here by "most" we mean an everywhere dense G_δ subset of the space of vector fields (cf., for instance, [4, 5, 8-10, 12]). In [2] we considered a class of vector fields which are Lipschitz on bounded subsets. We assumed there that the convex function f has a unique point of minimum, and moreover, that the minimization problem for the function f on X is well-posed. In the present paper, we obtain a generic convergence result for a class of vector fields which are only locally Lipschitz and bounded on bounded subsets of X, and for a convex function f which has a (not necessarily unique) point of minimum. We equip the space of these vector fields with a natural complete metric and show that the function f tends to its minimum along any trajectory of the dynamical system determined by a generic pair consisting of an initial condition and a vector field.