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# RETARDED AND MIXED NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH CONTINUOUS DELAY

T. Candan[1] and R. S. Dahiya[2]

[1]Department of Mathematics, Faculty of Art and Science
Niğde University, Niğde, 51200, Turkey.
email: tcandan@nigde.edu.tr
[2]Department of Mathematics
Iowa State University, Ames, IA 50011, U.S.A.
email: rdahiya@iastate.edu

**Abstract.** We obtain certain theorems to establish oscillation criteria for the arbitrary order neutral functional differential equation

$$\left[r(t)[x(t)+\int_a^b p(t,\mu)x(\tau(t,\mu))d\mu]^{(n-1)}\right]'+\int_c^d q\left(t,\xi\right)f\left(x\left(\sigma\left(t,\xi\right)\right)\right)d\xi=0.$$

**Keywords.** Oscillation, Neutral equations, Retarded differential equations, Nonlinear, Functional differential equations.
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# 1 Introduction

In this paper we are concerned with n-th order nonlinear neutral differential equations with continuous deviating arguments

$$\left[r(t)[x(t)+\int_a^b p(t,\mu)x(\tau(t,\mu))d\mu]^{(n-1)}\right]'+\int_c^d q\left(t,\xi\right)f\left(x\left(\sigma\left(t,\xi\right)\right)\right)d\xi=0, \qquad (1)$$

where $t\geqslant 0$, $r(t)\in C^1([t_0,\infty),\mathbb{R})$, $r(t)>0$ and $\int^\infty \frac{dt}{r(t)}=\infty$, $p(t,\mu)\in C([t_0,\infty)\times[a,b],\mathbb{R})$, $0\leqslant P(t)=\int_a^b p(t,\mu)d\mu<1$, $\tau(t,\mu)\in C([t_0,\infty)\times[a,b],\mathbb{R})$, $\tau(t,\mu)\leqslant t$, and $\tau(t,\mu)\to\infty$ as $t\to\infty$ and $\mu\in[a,b]$, $q(t,\xi)\in C([t_0,\infty)\times[c,d],\mathbb{R})$ and $q(t,\xi)>0$, $f(x)\in C(\mathbb{R},\mathbb{R})$ and $xf(x)>0$ for $x\neq 0$, $\sigma(t,\xi)\in C^1([t_0,\infty)\times[c,d],\mathbb{R})$, $\xi\in[c,d]$.

A solution $x(t)\in C([t_0,\infty),\mathbb{R})$ of (1) is called oscillatory if $x(t)$ has arbitrarily large zeros in $[t_0,\infty)$, $t_0>0$. Otherwise $x(t)$ is called nonoscillatory. Recently, the following type of equations have been studied

$$y^{(n)}(t)+q(t)f(y(\sigma(t)))=0,$$

see Abu-Kaff & Dahiya [1], and Olah [12] and [13]. Then, these results are extended to a more general equation

$$\left[a(t)[x(t)+p(t)x(\tau(t))]^{(n-1)}\right]'+\delta q(t)f(x(\sigma(t)))=0.$$