RETARDED AND MIXED NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH CONTINUOUS DELAY

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Abstract. We obtain certain theorems to establish oscillation criteria for the arbitrary order neutral functional differential equation
\[ r(t)x(t) + \int_{a}^{b} p(t,\mu)x(\tau(t,\mu))d\mu^{(n-1)} + \int_{c}^{d} q(t,\xi)f(x(\sigma(t,\xi)))d\xi = 0, \]
where \( t \geq 0, \) \( r(t) \in C^{1}([t_0,\infty), \mathbb{R}), \) \( r(t) > 0 \) and \( \int_{t_0}^{\infty} \frac{dt}{r(t)} = \infty, \) \( p(t,\mu) \in C([t_0,\infty) \times [a,b], \mathbb{R}), \) \( 0 \leq P(t) = \int_{a}^{b} p(t,\mu)d\mu < 1, \) \( \tau(t,\mu) \in C([t_0,\infty) \times [a,b], \mathbb{R}), \) \( \tau(t,\mu) \leq t, \) and \( \tau(t,\mu) \to \infty \) as \( t \to \infty \) and \( \mu \in [a,b], \) \( q(t,\xi) \in C([t_0,\infty) \times [c,d], \mathbb{R}) \) and \( q(t,\xi) > 0, \) \( f(x) \in C(\mathbb{R},\mathbb{R}) \) and \( f(x) > 0 \) for \( x \neq 0, \) \( \sigma(t,\xi) \in C^{1}([t_0,\infty) \times [c,d], \mathbb{R}), \) \( \xi \in [c,d]. \)


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1 Introduction

In this paper we are concerned with n-th order nonlinear neutral differential equations with continuous deviating arguments
\[ r(t)x(t) + \int_{a}^{b} p(t,\mu)x(\tau(t,\mu))d\mu^{(n-1)} + \int_{c}^{d} q(t,\xi)f(x(\sigma(t,\xi)))d\xi = 0, \]
where \( t \geq 0, \) \( r(t) \in C^{1}([t_0,\infty), \mathbb{R}), \) \( r(t) > 0 \) and \( \int_{t_0}^{\infty} \frac{dt}{r(t)} = \infty, \) \( p(t,\mu) \in C([t_0,\infty) \times [a,b], \mathbb{R}), \) \( 0 \leq P(t) = \int_{a}^{b} p(t,\mu)d\mu < 1, \) \( \tau(t,\mu) \in C([t_0,\infty) \times [a,b], \mathbb{R}), \) \( \tau(t,\mu) \leq t, \) and \( \tau(t,\mu) \to \infty \) as \( t \to \infty \) and \( \mu \in [a,b], \) \( q(t,\xi) \in C([t_0,\infty) \times [c,d], \mathbb{R}) \) and \( q(t,\xi) > 0, \) \( f(x) \in C(\mathbb{R},\mathbb{R}) \) and \( f(x) > 0 \) for \( x \neq 0, \) \( \sigma(t,\xi) \in C^{1}([t_0,\infty) \times [c,d], \mathbb{R}), \) \( \xi \in [c,d]. \)

A solution \( x(t) \in C([t_0,\infty), \mathbb{R}) \) of (1) is called oscillatory if \( x(t) \) has arbitrarily large zeros in \([t_0,\infty), \) \( t_0 > 0. \) Otherwise \( x(t) \) is called nonoscillatory. Recently, the following type of equations have been studied
\[ y^{(n)}(t) + q(t)f(y(\sigma(t))) = 0, \]
see Abu-Kaff & Dahiya [1], and Olah [12] and [13]. Then, these results are extended to a more general equation
\[ [a(t)x(t) + p(t)x(\tau(t))]^{(n-1)} + \delta q(t)f(x(\sigma(t))) = 0. \]