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A MULTISTEP ALGORITHM FOR ODEs

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Dedicated to Professor Tiberiu Coloşi on his seventieth anniversary

Abstract. The objective of this paper is to prove the convergence of a linear implicit multi-step numerical method for ordinary differential equations. The algorithm is obtained via Taylor approximations. The convergence is proved following the Dahlquist theory. As an additional topic, the time stability is established too. Comparative tests between some of the most known numerical methods and this method are presented.
Keywords. Taylor approximation, multi-step method, stability, consistency, time regions stability.

AMS (MOS) subject classification: 65L06, 65P99.

1 Introduction

We present a linear implicit m-step method LIL (Local Iterative Linearization) and prove its convergence applied for the following initial value problem

$$\dot{x} = f(t, x), \quad x(t_0) = x_0, \tag{1.1}$$

where $f : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^n$, $T > 0$, $t_0 \in \mathbb{R}_+$, is a C^m smooth Lipschitz function[1].

Although the classical linear multi-step algorithms are very known and utilized, the LIL characteristics (convergence properties, time stability and applications results) show that this numerical method could be considered as an interesting alternative to the widely used formulas.

The backward approximation of derivatives implies null coefficients of the odd order derivatives which represent a major advantage for the propagation of errors.

As a comparative test two simple ODEs with known analytical solutions and a chaotic continuous-time dynamical system, first studied by Fabrikant and Rabinovich [6] and recent numerically re-examined by Danca and Chen [3], was integrated using the LIL algorithm and some of the most known algorithms. The complex dynamic of this special model represented a real challenge for almost all of these methods as shown in Sect.5.

[1] The Lipschitz condition is necessary for the stability proof.