Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 769-786

ON STRUCTURE OF POSITIVE SOLUTIONS OF SINGULAR BOUNDARY VALUE PROBLEMS FOR IMPULSIVE DIFFERENTIAL EQUATIONS[1]

Yansheng Liu[a] and Xinzhi Liu[b]

[a]Department of Mathematics, Shandong Normal University
Jinan, 250014, P. R. China, E-mail: ysliu6668@sohu.com
[b]Department of Applied Mathematics, University of Waterloo Waterloo
Waterloo, Ontario N2L 3G1 Canada, E-mail: xzliu@uwaterloo.ca

Abstract. This paper investigates the structure of positive solutions of singular boundary value problems(BVP) for impulsive differential equations. The results obtained are the existence of an unbounded continuum of positive solution set and the behavior of solutions according to parameter λ. As a corollary, BVP has at least one positive solution for every $\lambda > 0$.
Keywords: Global structure, Continuum, Singular boundary value problem, impulsive, Positive solution.
AMS(Subject Classification): 34B15.

1 Introduction and some preliminaries

Consider the structure of positive solutions of singular boundary value problem for the following second order impulsive differential equation

$$\begin{cases} u^{''}(t) + \lambda f(t, u(t)) = 0, \quad t \in (0,1), \quad t \neq t_1; \\ \Delta u|_{t=t_1} = I(u(t_1)); \\ \alpha u(0) - \beta u'(0) = 0; \\ \gamma u(1) + \delta u'(1) = 0, \end{cases} \tag{1.1$_\lambda$}$$

where the parameter $\lambda \in \mathbf{R}^+ = [0, +\infty)$, $\alpha, \beta, \gamma, \delta \geq 0$, $\beta\gamma + \alpha\delta + \alpha\gamma > 0$; $f \in C[(0,1) \times (0, +\infty), \mathbf{R}^+]$, $f(t, u)$ may be singular at $t = 0, 1$ and $u = 0$.

When the impulse effects are absent, $(1.1)_\lambda$ reduces to singular boundary value problems(BVP, for short) for differential equations which has been studied extensively(see, for instance, [1-3], [10-11], [13-15] and references therein) since $(1.1)_\lambda$ often arises in the study of positive radial solutions of a nonlinear elliptic equation. when $f(t, x) = x^p$, $p > 0$, it is known as the generalized

[1]The Project Supported by the Rewarded Foundation of Outstanding Middle and Young Scientist of Shandong Province, and NSERC Canada.