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EXISTENCE, MULTIPLICITY AND NONEXISTENCE OF POSITIVE PERIODIC SOLUTIONS OF NONLINEAR INTEGRAL EQUATION ON THE INFINITE INTERVAL MODELLING INFECTIONS DISEASE

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Abstract. In this paper, by applying the well-known fixed point theorem in a cone, we consider a class of nonlinear integral equations and obtain the results on the existence of one positive periodic solution, two positive periodic solutions and nonexistence of positive periodic solutions.
Keywords. Positive periodic solutions; Nonlinear integral equation; Fixed point theorem; Fixed point index.
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1 Introduction

The existence problem of periodic solutions has been an interesting problem for a long time. We can find many pretty results on this problem by using the fixed point theorem. But to our knowledge, the results about several periodic solutions are far less.

As a model for the spread of certain infectious disease with periodic contact rate that varies seasonally, the nonlinear integral equation

$$x(t) = \int_{t-\tau}^{t} f(s, x(s))ds, \qquad t \in R \tag{1}$$

was studied by Cooke and Kaplan[1] and Leggett and Williams[2]. They obtained the existence results of nontrivial periodic nonnegative solution for (1) by Krasnoselskii's fixed point theorem in a cone.

In this paper, we discuss the following integral equation

$$x(t) = \lambda \int_{t-\tau(t)}^{t} K(t,s)f(s, x(s))ds, \qquad t \in R, \quad \lambda > 0. \tag{2}$$

When $\tau(t) \equiv \tau$ and $\lambda = 1$, (2) was studied by Agarwal and O'Regan[3] by using Krasnoselskii's fixed point theorem in a cone. In (2), x(t) represents the proportion of infectives in the population at time t, f(t,x(t)) is the proportion