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AN APPLICATION OF DEGREE THEORY TO A NONLINEAR BIHARMONIC EQUATION

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ABSTRACT: We investigate the multiplicity of solutions of the nonlinear biharmonic equation with Dirichlet boundary condition, $\Delta^2 u + c\Delta u = g(u)$ in Ω, where $c \in R$ and Δ^2 denotes the biharmonic operator. We reveal the multiplicity of solutions of the nonlinear biharmonic equation by degree theory.
Keywords. Dirichlet boundary condition, multiplicity of solutions, variational reduction method, eigenvalue, degree theory
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1 Introduction

Let Ω be a smooth bounded region in R^n with smooth boundary $\partial\Omega$. We study the multiplicity of solutions of the nonlinear biharmonic equation

$$\Delta^2 u + c\Delta u = g(u) \quad \text{in } \Omega, \tag{1.1}$$

$$u = 0, \quad \Delta u = 0 \quad \text{on } \partial\Omega,$$

where $c \in R$ and Δ^2 denote the biharmonic operator. Here we assume that $g : R \to R$ is a differentiable function such that $g(0) = 0$ and

$$g'(\infty) = \lim_{|u|\to\infty} \frac{g(u)}{u} \in R.$$

Let λ_k, $k \geq 1$ denote the eigenvalues and ϕ_k, $k \geq 1$ the corresponding eigenfunctions, suitably normalized with respect to $L^2(\Omega)$ inner product, of the eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in } \Omega,$$

$$u = 0 \qquad \text{on } \partial\Omega,$$

where each eigenvalue λ_k is repeated as often as its multiplicity. We recall that $0 < \lambda_1 \leq \lambda_2 \leq \cdots \to +\infty$, and that $\phi_1(x) > 0$ for $x \in \Omega$.

We state the main result of this paper.