

EXISTENCE RESULTS FOR SOME BVPs ASSOCIATED WITH SECOND ORDER ODEs

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Abstract. In this paper, we present new existence results for some Dirichlet boundary value problems associated with second order ordinary differential equations. We do not assume a Nagumo-Bernstein type condition on the nonlinearity. In fact, we obtain existence results for a new class of nonlinearities.

Keywords. BVP, Fixed point, Nagumo's condition.

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1 Introduction

1.1 The problem

In this paper, we are concerned with the following nonlinear Dirichlet boundary value problem:

$$(\mathcal{P}) \quad \begin{cases} u'' = f(t, u, u'), & 0 < t < 1 \\ u(0) = u(1) = 0 \end{cases} \quad (1.1)$$

where the function $f: [0, 1] \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous. Under a growth condition on the nonlinearity f , we first prove an existence result. Assumptions (2.1), (2.2), (3.3), (4.1) and (4.4) are, as far as we know, new conditions. Assumption (2.1) encompasses both the case where f has either a sub-linear or a super-linear polynomial growth but not allow for combined growth types. This is developed Section 2. In Sections 3 and 4, a special attention is paid to the case where f is the sum of two functions with distinct growths: two existence results are thus obtained; the first one extends the one obtained in Section 2 while in the second one, we use a different growth assumption on the nonlinearity. Some examples illustrate our results and comparisons with already known results are provided.