

## APPROXIMATE CONTROLLABILITY OF SECOND-ORDER NEUTRAL STOCHASTIC EVOLUTION EQUATIONS

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**Abstract.** Approximate controllability results are established for a class of abstract second-order neutral stochastic evolution equations in a real separable Hilbert space. Applications to stochastic equations are provided to illustrate the abstract theory.

**Keywords.** Second-Order Stochastic Evolution Equation, Neutral Equation, Approximate Controllability.

**AMS (MOS) subject classification:** 34K30, 34F05, 60H10.

### 1 Introduction

The focus of this investigation is the approximate controllability problem for the class of abstract neutral semi-linear stochastic evolution equations of the form

$$\begin{aligned} d[x'(t) - f_1(t, x(t))] &= [Ax(t) + Bu(t) + f_2(t, x(t))] dt + g(t, x(t)) dW(t), \\ x(0) = \xi, \quad x'(0) = \varsigma, \quad 0 \leq t \leq T, \end{aligned} \quad (1)$$

in a real separable Hilbert space  $H$ , where the linear (possibly multi-valued) operator  $A : D(A) \subset H \rightarrow H$  is the infinitesimal generator of a strongly continuous cosine family on  $H$ ;  $W$  is a  $K$ -valued Wiener process with incremental covariance given by the nuclear operator  $Q$  defined on a complete probability space  $(\Omega, \mathfrak{F}, P)$  equipped with a normal filtration  $(\mathfrak{F}_t)_{t \geq 0}$ ;  $f_i : [0, T] \times H \rightarrow H$  ( $i = 1, 2$ ) and  $g : [0, T] \times H \rightarrow L_2(Q^{1/2}K; H)$  (where  $K$  is another real separable Hilbert space and  $L_2(Q^{1/2}K; H)$  will be defined later) are appropriate mappings;  $B : U \rightarrow H$  is a given mapping;  $u$  is an appropriate control function; and  $\xi$  and  $\varsigma$  are  $\mathfrak{F}_0$ -measurable  $H$ -valued random variables independent of  $W$ .

Stochastic partial differential equations (SPDEs) arise naturally in the mathematical modeling of various phenomena in the natural and social sciences [28], [29], [32]. The qualitative properties (existence, stability, controllability, invariant measures, etc.) for first-order semilinear SPDEs have received much attention (see [3], [4], [20], [21], [22], [13] and references therein).