Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **13 (2006)** 555-567

ITERATED RUNGE-KUTTA METHODS OF IMPLICIT DIFFERENTIAL-ALGEBRAIC EQUATIONS

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Abstract. The waveform relaxation methods based on Runge-Kutta methods for solving implicit differential-algebraic equations are proposed. Convergence of the iterative processes with particular attention to parallelism are investigated. The theoretical results are illustrated by a few numerical examples.
Keywords. Implicit differential-algebraic equations, Runge-Kutta methods, waveform relaxation, converge, parallelism.
AMS (MOS) subject classification: 65L05, 65R20

1 Introduction

We consider the implicit differential-algebraic equations (DAEs)

$$\begin{cases} f(t,\dot{y}(t),y(t),x(t))=0,\ y(t_0)=y_0, \\ g(t,y(t),x(t))=0,\ t\in[t_0,t_T], \end{cases} \tag{1}$$

where $y(t)\in R^m$ and $x(t)\in R^n$ are given. Functions f,g are sufficiently differentiable and we denote $f=(f_1,\cdots,f_m)^T$ and $g=(g_1,\cdots,g_n)^T$ in this paper. In order that System (1) is index one problem we suppose $\|(g_x(t,y(t),x(t)))^{-1}\|\le M$ in a neighbourhood of the exact solution. Further the initial values y_0 and x_0 are consistent with (1), i.e. $g(t_0,y(t_0),x(t_0))=0$, that System (1) has a unique solution. Many dynamical systems can be described by the above DAEs of index-1.

Denote the Jacobian matrices $K_f:=f_u(t,u,v,w), J_f:=-f_v(t,u,\ v,w)$, $P_f:=f_w(t,u,v,w), K_g:=g_u(t,u,v)$ and $J_g:=g_v(t,u,v)$. The pair of matrices $\{K_f,J_f+P_fJ_g^{-1}K_g\}$ is said to be a stable pair, if the eigenspectrum $\sigma(K_f,J_f+P_fJ_g^{-1}K_g)$ of the pencil $(J_f+P_fJ_g^{-1}K_g)-\lambda K_f$ is in the non-positive half-plane; that is, the characteristic equation $det(\lambda K_f-J_f-P_fJ_g^{-1}K_g)=0$ has only zeros in the non-positive half-plane. In the convergence analysis of iterative method for solving the numerical discretization of DAEs (1), the property of matrix pairs will play a central role.

A large class of numerical discretization of DAEs (1) can be implemented by Runge-Kutta (RK) methods. Let$Y_n=(Y_{ni})\in R^{sm}, X_n=(X_{ni})\in R^{sn}$,