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ONE UNIQUE SOLUTION OF AN INITIAL VALUE PROBLEM FOR NONLINEAR FIRST-ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS OF VOLTERRA TYPE IN BANACH SPACES

Lishan Liu[1], Yumei Xu[1], Yonghong Wu[2] and Peg-Foo Siew[2]

[1]Department of Mathematics, Qufu Normal University,
Qufu 273165, Shandong, People's Republic of China
lls@mail.qfnu.edu.cn, yumeixu@eyou.com
[2]Department of Mathematics and Statistics, Curtin University of Technology,
Box 6845, Perth, Australia,
yhwu@maths.curtin.edu.au, pfsiew@maths.curtin.edu.au

Abstract. In this paper, we study the well-posedness and solution of an initial value problem for the nonlinear first-order impulsive integro-differential equations of Volterra type in Banach spaces. By applying the Banach fixed point theorem, an existence and uniqueness theorem is developed under simple conditions. In addition, an explicit iterative approximation of the solution and an error estimate of the approximation sequence for the initial value problem are derived. The assumed conditions in the present theorems are easy to be verified.
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1 Introduction

The theory of impulsive differential equations has become an important area of investigation in recent years, because its structure has deep physical interpretation and is based on realistic mathematical models. However the corresponding theory for impulsive integro-differential equations in abstract spaces has yet to be developed (see [3, 5]).

In this paper, we consider the following initial value problem(IVP, in short) for first order nonlinear impulsive integro-differential equations of Volterra type in a real Banach space $(E, \|\cdot\|)$:

$$\begin{cases} x' = f(t, x, Tx), & \forall t \in J, t \neq t_k, \\ \Delta x|_{t=t_k} = I_k(x(t_k)), & k = 1, 2, 3, \cdots, m, \\ x(0) = x_0, \end{cases} \tag{1.1}$$