

STRONG CONVERGENCE THEOREMS FOR ASYMPTOTICALLY NONEXPANSIVE SEMIGROUPS IN GENERAL BANACH SPACES

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Abstract. We establish fixed point theorems and strong convergence theorems of Mann's type for asymptotically nonexpansive semigroups with compact domains in general Banach spaces.

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1 Introduction

In 1975, Baillon [3] originally studied the first nonlinear ergodic theorem in Hilbert space: Let C be a closed and convex subset of a Hilbert space H and let T be a nonexpansive mapping of C into itself, that is, $\|Tx - Ty\| \leq \|x - y\|$ for each $x, y \in C$. If the set $F(T)$ of fixed points of T is nonempty, then for each $x \in C$, the Cesàro means

$$S_n(x) = \frac{1}{n} \sum_{k=0}^{n-1} T^k x$$

converge weakly to some $y \in F(T)$. In this case, putting $y = Px$ for each $x \in C$, P is a nonexpansive retraction of C onto $F(T)$ such that $PT = TP = P$ and Px is contained in the closure of convex hull of $\{T^n x : n = 1, 2, \dots\}$ for each $x \in C$. Using the theory of means of abstract semigroups [6], Takahashi [25, 26] introduced the notion of "ergodic retractions" for semigroups of nonexpansive mappings and proved the existence of ergodic retractions for amenable semigroups of nonexpansive mappings in Hilbert spaces. Later, this result was fully extended to amenable semigroups of nonexpansive mappings in uniformly convex Banach spaces with Fréchet differentiable norm by Lau, Shioji and Takahashi [12]; see also [8].

On the other hand, Shimizu and Takahashi [18, 19] introduced the first iterative schemes for finding common fixed points of families of nonexpansive mappings and proved strong convergence theorems for discrete and one-parameter semigroups of nonexpansive mappings in Hilbert spaces. Since