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OPTIMAL FILTERING FOR LINEAR SYSTEMS WITH MULTIPLICATIVE AND ADDITIVE WHITE NOISES

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Abstract. The optimal filtering problem is solved for the system described by the continuous, linear, n-dimensional ordinary differential equation with multiplicative and additive Gaussian white noises. Two different forms of the optimal filter are derived. The first form is most suitable if the optimal estimates of only a few derivatives of order 0 to $(n-1)$ are of interest.

Keywords. filtering, stochastic system, multiplicative and additive white noises
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1 Introduction

The Kalman filter (KF) [5] solves the problem of the optimal state estimation of linear state space system with additive white noise. The filtering and control for systems, subject to both additive and multiplicative noises, are the problems of practical importance, for which the traditional KF is not applicable.

The problem with multiplicative noises arises naturally if the intensity of the stochastic disturbances depends on the state of the system and, therefore, is unknown *a priori*. Examples of linear systems with multiplicative noises can be found in signal processing [12], nonlinear mechanics [9], bimolecular interactions [2], and aerospace engineering systems [7]. Several approaches have been proposed to estimate the states of linear systems with multiplicative noises, including linear matrix inequality approach [4, 8], Riccati difference equation method [13], and game theoretic approach [10].

In this paper, two forms of the optimal, in the Kalman sense, filter for the continuous, stochastic ODE systems with additive and multiplicative noises are derived. The first optimal filter is obtained by formulating the problem as the optimal state estimation for the general Itô-Volterra system, for which the optimal filtering theory has recently been developed [1, 14]. The resulting optimal filter is given by two coupled scalar integral equations, both of which must be solved in real time to generate the optimal estimate of the $(n-1)$ order derivative of the ODE system state. It is then shown that the optimal state estimate of any derivative of order 0 to $(n-2)$ can