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DISCRETE ADMISSIBILITY, ℓ^p-SPACES AND EXPONENTIAL DICHOTOMY ON THE REAL LINE

Adina Luminiţa Sasu[1] and Bogdan Sasu[1]

[1]Department of Mathematics
Faculty of Mathematics and Computer Science
West University of Timişoara, Bul. V. Pârvan No. 4
300223-Timişoara, Romania

Abstract. The aim of this paper is to obtain characterizations in terms of ℓ^p-spaces for exponential dichotomy of evolution families on the real line. We show that if the pair $(\ell^p(\mathbb{Z}, X), \ell^q(\mathbb{Z}, X))$ is admissible for a discrete evolution family $\Phi = \{\Phi(m, n)\}_{m \geq n}$ then Φ is uniformly exponentially dichotomic. We prove that the condition of $(\ell^p(\mathbb{Z}, X), \ell^q(\mathbb{Z}, X))$-admissibility becomes necessary for uniform exponential dichotomy if and only if $p \geq q$. After that, we apply the results for general evolution families and we obtain conditions for uniform exponential dichotomy using the admissibility of the pair $(\ell^p(\mathbb{Z}, X), \ell^q(\mathbb{Z}, X))$.
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1 Introduction

Discrete time methods have a significant role in the study of the exponential dichotomy of evolution equations (see [1], [3], [5], [11], [12], [14]-[18]). These approaches have the starting point in Henry's result which connects the uniform exponential dichotomy of a sequence of bounded linear operators $(T_n)_{n \in \mathbb{Z}}$ with the unique solvability in $\ell^\infty(\mathbb{Z}, X)$ of the discrete equation $x_{n+1} = T_n x_n + f_n$ (see [5]). An important step has been made in [3], where the authors introduced the concept of discrete dichotomy for a skew-product sequence and generalized Henry's theorem for this case. Generally, the exponential dichotomy of evolution families on the half-line may be characterized in terms of the solvability of a discrete or integral associated equation, in the hypothesis that the space of the initial conditions is closed and complemented (see [11], [13], [18]). Discrete-time characterizations for uniform exponential dichotomy in terms of the admissibility of the pair $(c_0(\mathbb{N}, X), c_{00}(\mathbb{N}, X))$ have been obtained in [11], for the case of evolution families on the half-line. The techniques in [11] have been extended for ℓ^p-spaces in [18], where uniform exponential dichotomy of an evolution family $\mathcal{U} = \{U(t, s)\}_{t \geq s \geq 0}$, on the half-line, in a Banach space X, was characterized in terms of the admissibility of the pair $(\ell^p(\mathbb{N}, X), \ell_0^q(\mathbb{N}, X))$, under the assumption that the subspace $X_1 = \{x \in X : U(\cdot, 0)x \in \ell^p(\mathbb{N}, X)\}$ is closed and complemented in