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ASYMPTOTIC BEHAVIOR OF SOLUTIONS FOR A CLASS OF SYSTEMS OF DELAY DIFFERENCE EQUATIONS

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Abstract. Consider the system of delay difference equations

$$\begin{cases} x_n - x_{n-1} &= -F(x_n) + G(y_{n-l}), \\ y_n - y_{n-1} &= -F(y_n) + G(x_{n-k}), \end{cases} \quad n = 1, 2, \cdots ,$$

where k and l are positive integers, F, $G \in C(R^1)$, and F is nondecreasing. It is shown that if $F(x) \geq G(x)$ for all $x \in R^1$ (or $F \leq G$ for all $x \in R^1$), then every bounded solution of such a class of systems tends to a constant vector. Our results improve and extend some corresponding ones already known.
Keywords. Asymptotic behavior, bounded solution, constant vector, delay difference system, positive limit set.
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1 Introduction

In this paper, we consider the following difference system with delays

$$\begin{cases} x_n - x_{n-1} &= -F(x_n) + G(y_{n-l}), \\ y_n - y_{n-1} &= -F(y_n) + G(x_{n-k}), \end{cases} \quad n = 1, 2, \cdots , \tag{1.1}$$

where k and l are positive integers, $F, G \in C(R^1)$, and either $F(x) \geq G(x)$ for all $x \in R^1$ or $F(x) \leq G(x)$ for all $x \in R^1$. Such a system can be regarded as a discrete analogue version of the following differential system with delays:

$$\begin{cases} x'(t) &= -F(x(t)) + G(y(t-\tau_1)), \\ y'(t) &= -F(y(t)) + G(x(t-\tau_2)), \end{cases} \quad t \geq 0, \tag{1.2}$$

where τ_1 and τ_2 are positive constants. The system (1.2) as models of various phenomena have been the subject of intensive studies in recent years (see, for example, [2, 3, 10]). In particular, if $\tau_1 = \tau_2 = r$ and we consider the synchronized solutions of (1.2) with $x(t) = y(t) = \varphi(t)$ for $t \in [-\max\{\tau_1, \tau_2\}, 0]$, then we get the following delay differential equations:

$$x'(t) = -F(x(t)) + G(x(t-r)). \tag{1.3}$$

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