

## INDIVIDUAL STABILITY FOR EVOLUTIONARY PROCESSES

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**Abstract.** We study the asymptotic behaviour of an individual orbit  $U(\cdot, t_0)x_0$ , of an evolutionary process  $\mathcal{U} = \{U(t, s)\}_{t \geq s \geq 0}$  with fixed  $t_0$  and  $x_0$ . We establish connections between the exponential stability of the above orbit and the admissibility of some pairs of Schäffer spaces.

**Keywords.** evolutionary processes, individual exponential stability

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### 1 Introduction

The problem of input-output stability was firstly studied by O. Perron in 1930 [15] for the case of linear finite-dimensional continuous-time systems  $x'(t) = A(t)x(t) + f(t)$ . In his paper, a central concern is the relationship, for linear equations, between the condition that the non-homogenous equation has the some bounded solution for every bounded "second member" on the one hand and a certain form of stability of the solution of the homogenous equation on the other.

After this seminal researches, relevant results concerning the extension of Perron's problem in the more general framework of infinite-dimensional Banach spaces were obtained by M. G. Krein [5], J. L. Daleckij [5], R. Bellman [3], J. L. Massera [9] and J. J. Schäffer [9]. In the last three decades this type of results has been developed extensively on the case of evolutionary processes in the papers due to N. van Minh [10,11,12], F. Rabiger [10], Y. Latushkin [4,7,8,] P. Randolph [7,8], R. Schnaubelt [8,10] and the achieved results have found many applications in the theory of partial differential equations.

The concept of the exponential stability from these papers is in fact equivalent with the exponential stability for each orbit  $U(\cdot, t_0)x$  of the evolutionary process  $\mathcal{U} = \{U(t, s)\}_{t \geq s \geq 0}$ .

Recently, increasing interest has been observed for the asymptotic behaviour of a single orbit  $T(\cdot)x_0$  of a  $C_0$ -semigroup  $\{T(t)\}_{t \geq 0}$ . We can mention here the papers due to J. van Neerven, C. J. K. Batty, F. Rabiger, Vu Quoc