

## A Note on Mean-Field Theory for Scale-Free Random Networks<sup>1</sup>

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**Abstract.** Usually, the BA model describes the growth and preferential attachment of scale-free networks. In this paper we present the G growth model and the Poisson model in order to describe real networks more precisely than the BA model. We recognize that the vertices arrival process is a renewal process. By using the renewal process theory and continuum theory, we calculate the degree distribution and stationary average degree distribution of the models. The consequences are that the stationary average degree distributions of these models are independent of the arrival process of vertices of the networks and the degree distributions are dependent on the arrival process. The advantage of our results is more accurate than that reported in [1, 2, 3]. Furthermore, the flaw of analysis in [1, 2, 3] is modified in this paper.

**Keywords.** BA model, Poisson model, G growth model, complex network, degree distribution, renewal process.

## 1 Introduction

The study of complex networks is getting intensive. There are many works focusing on the formulation of models of complex networks (see [1, 6]). Among them, the BA model is a well-known model and it is a network growth model. Especially, the BA model assumes that the vertices enter the network at equal time intervals, and the proofs in [1, 2, 3] used this assumption. We believe that the vertices arrival process should be a random process because the latter can describe many real networks more precisely. Secondly, we found a flaw in the analysis in the BA model (see [1, 2, 3]) because it led to a contradiction. Motivated by our observation, first of all, we present two new growth network models (the G growth network and the Poisson model), in which the arrival process of vertices can be a renewal process because we believe that the G growth network and the Poisson model can describe real networks more precisely. Furthermore, we found a flaw in the analysis in

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<sup>1</sup>This note focuses mainly on paper [3] entitled: Mean-field theory for scale-free random networks.