Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **13 (2006)** 497-504
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LIMIT OF PAGERANK WITH DAMPING FACTOR

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Abstract. PageRank is defined as the invariant probability of a Markov chain, whose transition matrix is induced by perturbing a web graph with a damping factor α. In this paper, we discuss the limiting behavior of PageRank when α tends to 1. We show that the limit is unique. We also give an analytic representation of the limit.

Keywords. PageRank, damping factor, Markov chain, invariant probability, search engine.

AMS (MOS) subject classification: 60J27, 60F99, 68W99.

1 Introduction

PageRank is one of the most important ranking algorithm used by search engines. Its notable advantages are that it is query independent, content independent, and only dependents on the web structure. In practice, it is a simple, effective and robust search method on networks [3][5][8][9].

PageRank is defined as the invariant probability of a Markov chain, which measures the relative importance of webpages. This chain is obtained by perturbing the adjacent matrix of a web graph with a damping factor α, which spreads uniformly part of the ranking. In details, the algorithm of PageRank is described as follows [3][5][8][9].

Consider the hyperlink structure of the webpages on a network as a directed graph, $G = (V(G), E(G))$. A vertex $i \in V(G)$ of the graph represents a webpage and a directed edge $\overrightarrow{ij} \in E(G)$ represents a hyperlink from the webpage i to j. Let $\mathbf{A}$ be the adjacent matrix of G and b_i be the sum of the ith row of $\mathbf{A}$. Let $\mathbf{B}$ be the diagonal matrix with diagonal entries b_i (if $b_i = 0$, then we normalize $b_i = N$, the cardinal number of $V(G)$). Now, we construct a Markov transition matrix $\mathbf{P} = \mathbf{B}^{-1} \cdot \mathbf{A}$. We may naively imagine that a surfer surfs on the Internet and chooses the next page by *randomly* clicking at one of the links in the current page (*i.e.* with uniform probability). This means that the surfer randomly walk on G with transition probability $\mathbf{P}$. The more important a webpage is, the higher frequency it will be visited. So,