

## EVOLUTIONARY DYNAMICS OF THE COURNOT GAME MODEL ON SMALL-WORLD NETWORKS

Wu Jia<sup>1</sup>, Lixin Ding<sup>2</sup> and Wentao Fan<sup>3</sup>

<sup>1</sup>Institute of Systems Engineering  
Wuhan University, Wuhan, 430072, P. R. China

<sup>2</sup>State Key Laboratory of Software Engineering  
Wuhan University, Wuhan, 430072, P. R. China

<sup>3</sup>Wuhan Institute of Physics and Mathematics  
Chinese Academy of Sciences, Wuhan, 430071, P. R. China

**Abstract.** A model of learning on small-world networks is built in the context of Cournot oligopoly. The difference in the evolutionary dynamics of the model for different rewiring probabilities of the networks is analyzed. Some expressions for the evolutionary rules are presented. Simulation results supporting cooperations among firms are obtained and the  $R_0$ -return time of the pre-equilibrium state of the model is explored, which exhibits distinctive dynamical behaviors of the model on small-world networks. Some variants to this model are also considered and their dynamical behaviors are compared.

**Keywords.** Cournot duopoly game,  $R_0$ -return time, cooperative equilibrium, mutation, pre-equilibrium state.

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### 1 Introduction

The cooperative behavior is something that is frequently observable in many economic and social contexts, which traditional game theory fails to explain. It is shown that the cooperative behavior is the possible outcome of repeated interactions among bounded rational agents that change their actions following some adaptive behavioral rules [1], and that the probability of the evolution of the system towards a cooperative equilibrium is greatly increased if a form of localization of the players is assumed [2, 3]. A lot of papers on evolutionary games and learning have explicitly studied the implications of local interactions in games [4, 5]. Most of the models in the literature have been studied on regular networks. However, as argued by Watts and Strogatz [6], most real networks are neither perfectly ordered nor completely disordered but fall under the category between the two extremes. The topology of social and economic communities is much better described by what has been called small-world networks [7]. Such networks are characterized by a high degree of local order, yet appear disordered on a large scale because of the presence of shortcuts in the networks. Aellen and Ellison illustrated that the changes