

## SYNCHRONIZATION OF PERIODIC TRAJECTORIES FOR LINEARLY COUPLED MAP LATTICES WITH DELAYED COUPLING

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**Abstract.** Synchronization of periodic trajectories of linearly coupled map lattices with delayed coupling is investigated. A quantity  $d$  involving the spectra of coupling matrix and the dynamics of an individual node is introduced to analyze the stability of the synchronized periodic trajectory. A sufficient criterion guaranteeing synchronization and de-synchronization of the periodic trajectory is obtained. Dependence of the stability of the synchronized trajectory on the coupling delay is also revealed.

**Keywords.** Complex dynamical system, delayed coupling, periodic solution, stability, synchronization.

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### 1 Introduction

Linearly Coupled Map Lattices (LCMLs) constitute a large class of dynamical systems with discrete space and time, as well as continuous state (see [4], [6]). The coupled system can be modelled as follows:

$$x_i(t+1) = f(x_i(t)) + \frac{\epsilon}{k_i} \sum_{j \neq i, j=1}^m b_{ij} [f(x_j(t)) - f(x_i(t))]$$

where  $t \in N$ ,  $x_i(t)$  denotes the state value of node  $i$ ,  $i = 1, 2, \dots, m$ ,  $f(\cdot)$  is a continuous function,  $b_{ij} \geq 0$ ,  $k_i = \sum_{j \neq i} b_{ij}$ , and  $\epsilon$  is the coupling strength.

In many biological and physical systems, coupling delay occurs among nodes in the networks (see [1]). Then, the following LCMLs with coupling delay is considered:

$$x_i(t+1) = f(x_i(t)) + \frac{\epsilon}{k_i} \sum_{j \neq i, j=1}^m b_{ij} \left[ f(x_j(t-\tau)) - f(x_i(t)) \right] \quad (1)$$

where  $\tau$  is the coupling delay.