ON TOPOLOGICAL DEGREE THEORY FOR MAPPINGS OF CLASS \((LS_+)\) IN REFLEXIVE BANACH SPACES

Yu Qing Chen\(^1\) and Yeol Je Cho\(^2\)

\(^1\)Department of Mathematics
Foshan University
Foshan, Guangdong 528000, P. R. China
\(^2\)Department of Mathematics Education and the RINS
College of Education
Gyeongsang National University
Chinju 660-701, Korea

Abstract. In this paper, we introduce the mapping of class \((LS_+)\) in reflexive Banach spaces, which is a generalization of the mapping of class \((S_+)\), and then we construct a degree theory for mappings of class \((LS_+)\).

Key Words: Brouwer’s degree theory, Leray-Schauder degree, monotone and maximal monotone operators, mappings of classes \((S_+)\), \((S_+)\), and \((LS_+)\), \(L\)-pseudo-compact mapping, homotopy of mappings of class \((LS_+)\), pseudo-monotone and generalized pseudo-monotone mappings, topological degree.

AMS (MOS) Subject Classification: 1991 Mathematics Subject Classification: 47H09, 47H10, 49J30, 47H06.

1 Introduction and Preliminaries

In 1934, Leray and Schauder generalized Brouwer’s degree theory to an infinite Banach space and established the so called Leray-Schauder degree. It turns out that the Leray-Schauder degree is a very powerful tool in proving various existence results for nonlinear partial differential equations. So there rises a very natural problem: For what kind of mappings, can we establish a degree theory? Many people has studied this problem, degree theories for non-compact mappings, such as \(k\)-set contraction mapping, \(A\)-proper mapping, mappings of class \((S_+)\), \((S_+)\), and \((LS_+)\), \(L\)-pseudo-compact mapping, homotopy of mappings of class \((LS_+)\), pseudo-monotone and generalized pseudo-monotone mappings, topological degree.

2 The corresponding author: yjchogsnu.ac.kr (Yeol Je Cho).
3The second author was supported from the Korea Research Foundation Grant (KRF-2004-041-C00033).