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ON THE GLOBAL WEAK SOLUTIONS TO AN INTEGRABLE SHALLOW WATER EQUATION WITH LINEAR AND NONLINEAR DISPERSION

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Abstract. We prove the existence and uniqueness of global weak solutions for an integrable shallow water equation that describes the unidirectional nonlinear wave equation combining the linear dispersion of the KdV equation with the nonlinear/nonlocal dispersion of the Camassa-Holm equation.
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1 Introduction

Recently, Dullin, Gottwald, and Holm [16] derived a new equation describing unidirectional water waves with fluid velocity $u(x,t)$:

$$\begin{cases} u_t - \alpha^2 u_{txx} + c_0 u_x + 3uu_x + \gamma u_{xxx} = & \alpha^2(2u_x u_{xx} + uu_{xxx}), \\ & t > 0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}. \end{cases} \tag{1.1}$$

Here the constants α^2 and $\frac{\gamma}{c_0}$ are squares of length scales, and c_0 is a nonnegative parameter related to the linear wave speed in shallow water, and $u(t,x)$ stands for the fluid velocity. The equation, which is derived by the method of asymptotic analysis and a near-identity normal form transformation from water wave theory, combines the linear dispersion of the KdV equation with the nonlinear/nonlocal dispersion of the Camassa-Holm equation. It is completely integrable [16].

With $\alpha = 0$ in Eq.(1.1) we find the well-known Korteweg-de Vries equation which describes the unidirectional propagation of waves at the free surface of shallow water under the influence of gravity. $u(t,x)$ represents the wave height above a flat bottom, x is proportional to distance in the direction of propagation and t is proportional to elapsed time. The Cauchy problem of