Dynamics of Continuous, Discrete and Impulsive Systems
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INFINITE-DIMENSIONAL DYNAMICAL SYSTEMS INDUCED BY INTERVAL MAPS[1]

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Abstract. In the present paper, we consider the infinite dimensional dynamical system

$$(C([0,\ 1],\ I),\ F), \qquad F(\varphi) = f \circ \varphi \ \text{ for } \ \varphi \in C([0,\ 1],\ I),$$

which is induced by a continuous interval map f from I into itself. The fluctuation orbits of this infinite dimensional dynamical system are classified in terms of growth rates of the total variations of iterates of the corresponding interval map. As an application, the fluctuation solutions of one-dimensional wave equation with a Van der Pol boundary condition are considered.

Keywords: dynamical system, chaos, total variation, complexity, wave equation, homoclinic point.
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1 Introduction

Let I be a closed interval in $\mathbb{R}$, $C([0,\ 1],\ I)$ be the space of continuous functions from $[0,\ 1]$ to I and f be a continuous map from I into itself. That is, $(I,\ f)$ is an one-dimensional continuous discrete dynamical system. In this paper, we consider the infinite-dimensional discrete dynamical system (I3DS)

$$(C([0,\ 1],\ I), F), \qquad F(\varphi) = f \circ \varphi \ \text{ for } \ \varphi \in C([0,\ 1],\ I), \tag{1.1}$$

which is induced by the one-dimensional system $(I,\ f)$.

The motivation of studying the I3DS (1.1) is from the continuous time difference equation

$$x(t+1) = f(x(t)), \qquad t \in \mathbb{R}^+, \tag{1.2}$$

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