Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 499-507

SOME SYSTEMS OF NONLINEAR DIFFERENCE EQUATIONS OF HIGHER ORDER WITH PERIODIC SOLUTIONS

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Abstract. We show that every positive solution of the system of difference equations

$$x_{n+1}^{(1)} = \frac{1 + x_n^{(2)}}{x_{n-1}^{(3)}}, \quad x_{n+1}^{(2)} = \frac{1 + x_n^{(3)}}{x_{n-1}^{(4)}}, \ldots, \; x_{n+1}^{(k)} = \frac{1 + x_n^{(1)}}{x_{n-1}^{(2)}}, \quad n \in \mathbb{N}_0,$$

where $k \in \mathbb{N}$ is fixed, is periodic with period equal to $5k$ if $k \not\equiv 0 \pmod 5$, and with period k if $k \equiv 0 \pmod 5$. It is shown also that every positive solution of the system of difference equations

$$x_{n+1}^{(1)} = \frac{1 + x_n^{(2)} + x_{n-1}^{(3)}}{x_{n-2}^{(4)}}, \quad x_{n+1}^{(2)} = \frac{1 + x_n^{(3)} + x_{n-1}^{(4)}}{x_{n-2}^{(5)}}, \ldots, \; x_{n+1}^{(k)} = \frac{1 + x_n^{(1)} + x_{n-1}^{(2)}}{x_{n-2}^{(3)}},$$

$n \in \mathbb{N}_0$, is periodic with period equal to $2^{3-i}k$, if $GCD(k, 8) = 2^i$, $i \in \{0, 1, 2, 3\}$ (the greatest common divisor of k and 8). Two more systems are considered. These results generalize the well-known periodicity of the corresponding scalar equations.

Keywords. System of difference equations, periodicity, positive solution.
AMS (MOS) subject classification: Primary 39A20.

1 Introduction and preliminaries

Although recently there has been a great interest in studying the behavior of the solutions of rational nonlinear difference equations, there are only a few papers devoted to systems of rational nonlinear difference equations or difference equations with maximum, mostly to systems of two difference equations of order one, see, for example, [6] and the references therein. Some global convergence results concerning nonlinear systems can be found in [7].

It is well known that all well defined solutions of the difference equation

$$x_{n+1} = \frac{1 + x_n}{x_{n-1}}, \quad n \in \mathbb{N}_0, \tag{1}$$

are periodic with period five. Equation (1) is attributed to Lyness ([2, 3, 4, 5]) and it arises in frieze patterns.