Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 465-472

THE INTERACTION OF PEAKONS AND ANTIPEAKONS

Erik Wahlén[1]

[1]Department of Mathematics
Lund University, PO Box 118, 22100 Lund, Sweden

Abstract. In this note we discuss some aspects of the peakon-antipeakon interaction for the Camassa-Holm equation.
Keywords. Shallow water equation, Weak solutions, Solitons, Hamiltonian system, Wave breaking.
AMS (MOS) subject classification: 35L05, 35Q35.

1 Introduction

The equation

$$\begin{cases} u_t - u_{txx} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, & t > t_0, \quad x \in \mathbb{R}, \\ u(t_0, x) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1.1}$$

was discovered by Fokas and Fuchssteiner [16] as a bi-Hamiltonian system with infinitely many conservation laws. It was later obtained as a model for shallow water waves, $u(t,x)$ representing in nondimensional variables the water's free surface above a flat bottom, by Camassa and Holm [2] (see also [17] for a discussion of the physical relevance of (1.1) in the context of shallow water waves). Among the other interesting aspects of (1.1), we would like to point out that it is completely integrable [4, 10] and that it is a reexpression of geodesic flow on the diffeomorphism group of the line (see [8, 9, 20]).

Equation (1.1) has solitary waves of the form

$$u_c(t,x) = c\varphi(x - ct), \quad x \in \mathbb{R},$$

where $\varphi(x) = e^{-|x|}$. We call u_c a *peakon* if $c > 0$ and an *antipeakon* if $c < 0$. These waves have peaked corners at $x = ct$, and therefore have to be interpreted as weak solutions, using the fact that (1.1) can be reformulated as the nonlocal conservation law

$$u_t + uu_x + \partial_x p * \left(u^2 + \frac{1}{2}u_x^2\right) = 0, \tag{1.2}$$

where $p(x) := \frac{1}{2}e^{-|x|}$.

The solitary waves are solitons, meaning that they retain their individuality under interaction and eventually emerge with their original shapes and