

BACKWARD EULER MIXED FEM AND REGULARITY OF PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS WITH NON-SMOOTH DATA

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Abstract. The non-Fickian flow of fluid in porous media is complicated by the history effect which characterizes various mixing length growth of the flow, and can be modeled by an integro-differential equation. This paper studies a backward Euler scheme for the mixed finite element approximate solution of such problems with non-smooth initial data. A new regularity result is derived for the model problem, which can be used to design high order numerical schemes in time .

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1 Introduction

The transport of contaminants in the subsurface is currently a very active research area in porous media flow modeling. The evolution of a reactive chemical within a velocity field exhibiting excitements on many scales typically represented using classical Fickian dispersion theory. The evolution in such velocity field, when modeled with Fickian type constitutive laws, leads to a dispersion tensor depending upon the timescales of observation. In order to avoid this difficulty, non-local Fickian models have proposed recently. This type of models can represent some important history or memory effects which deserves much attention both in theoretical analysis and numerical approximations.

Cushman and his colleagues [4] have developed a non-local theory and some applications for the flow of fluid in porous media. Furtado, Glimm, Lindquist and Pereira [16] and Glimm, Lindquist, Pereir, and Zhang [17] Neuman and Zhang [21], and Ewing [10] [11] [12] also studied the history effect of various mixing length growth for flow in heterogeneous porous media. In a recent laboratory experimental investigation of contaminant transport in heterogeneous porous media [24], some nonlocal behavior of dispersion tensors have been observed.