Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 231-246

SPREADING SPEED AND TRAVELING WAVES FOR THE DIFFUSIVE LOGISTIC EQUATION WITH A SEDENTARY COMPARTMENT

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Abstract. By applying the theory of asymptotic speeds of spread and traveling waves to the diffusive logistic equation with a sedentary compartment, we establish the existence of minimal wave speed for monotone traveling waves and show that it coincides with the spreading speed for solutions with initial functions having compact supports.

Keywords. Spreading speed; Traveling waves; The diffusive logistic equation; A sedentary compartment.
AMS (MOS) subject classification: 35B40, 35K57, 92D25

1 Introduction

It is well known that the diffusive logistic or Verhulst equation is a scalar reaction diffusion equation with a simple hump nonlinearity (quadratic nonlinearity in the classical case). This equation describes the immigration of a species into a territory or the advance of an advantageous gene into a population. The equation provides the classical example for traveling fronts in parabolic equations, and it forms the nucleus of more complex multi-species models in ecology, pattern formation and epidemiology (see, e.g., [7]). In order to consider the case where the population individuals switch between mobile and stationary states during their lifetime, Lewis and Schmitz [4] presented and analysed the following reaction-diffusion model

$$\begin{cases} \partial_t v = D\Delta v - \mu v - \gamma_2 v + \gamma_1 w, \\ \partial_t w = rw(1 - w/K) - \gamma_1 w + \gamma_2 v, \end{cases} \tag{1.1}$$

where $v(t,x)$ and $w(t,x)$ are spatial densities of migrating and sedentary subpopulations, respectively, D is diffusion coefficient of migrating subpopulation, γ_1 and γ_2 are transition rates between two states. In model (1.1), the migrants have a positive mortality μ while the sedentary subpopulation reproduces (with the intrinsic growth rate r) and is subject to a finite carrying

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