Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 209-220

Asymptotic Behavior of Three-Dimensional Ginzburg-Landau Type Equation

Shujuan Lü and Qishao Lu

Department of Mathematics

Beijing University of Aeronautics and Astronautics, Beijing 100083, P. R. China

Abstract. A complex Ginzburg-Landau type equation with periodic initial value condition in three spatial dimensions is considered. Sufficient conditions for existence and uniqueness of global solutions are obtained by a uniform priori estimates of solutions. Furthermore, the existence of global attractor with finite Hausdorff and fractal dimensions is proved.

Keywords. Ginzburg-Landau type equation, Global solution, Global attractor, Hausdorff dimension, Fractal dimensions.

AMS (MOS) subject classification: 35K57, 35B40

1 Introduction

The Ginzburg-Landau type equation is a kind of important nonlinear evolution equations, which are proposed in many mechanics, physical and chemistry problems. For this reason, the research of them is of theoretical and practice significance. There have been many discussions on the Ginzburg-Landau equation (GLE)

$$u_t - (1+\mathrm{i}\nu)\triangle u + (1+\mathrm{i}\mu)|u|^{2\sigma}u - \gamma u = 0, \tag{1.1}$$

and the generalized Ginzburg-Landau equation (GGLE)

$$u_t = \rho u + (1+\mathrm{i}\nu)\triangle u + (1+\mathrm{i}\mu)|u|^{2\sigma}u + \alpha\lambda_1\cdot\nabla(|u|^2u) + \beta(\lambda_2\cdot\nabla u)|u|^2, \tag{1.2}$$

in one or two spatial dimensions. For example, Ghidaglia and Héron [1] and Doering et al. [2], Promislow [3], Bu [4] and Lü[5], studied the finite-dimensional attractor and related dynamical issues for the 1D or 2D GLE (1.1) with the degree of nonlinearity $\sigma = 1$ or $\sigma = 2$. Guo et al. [6,7] and Gao [8,9] considered 2D generalized Ginzburg-Landau equation (1.2). They studied the existence of the global solution and the finite-dimensional global attractor of (1.2) with periodic boundary condition or with Cauchy condition. But the case of three spatial dimensions, relevant results are seldom. Doering, Gibbon and Levermore [10] considered the equation (1.1) in three-dimensional space and for the degree of nonlinearity $\sigma > 0$. They