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ASYMPTOTIC PROPERTIES OF A REVISED SIR EPIDEMIC MODEL WITH DENSITY DEPENDENT BIRTH RATE AND TIME DELAY

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Abstract. In this paper, based on some well known SIR epidemic models, we propose a class of revised SIR epidemic model with time delays. In the model, the birth rate of population depends on the density of population. With some classical analysis techniques for functional differential equation, we show that the model has very similar locally and globally dynamic properties as the models with density independent birth rate.
Keywords. SIR epidemic model, time delay, local asymptotic stability, global asymptotic stability.
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1 Introduction

Because of importance of biology meaning, dynamical properties of epidemic models with or without time delays and general theory on such dynamical systems have been studied by many authors (see, for example, [1]-[20], the recent monograph [21] and the references in it). Especially, it is shown that time delays play an important roles to the dynamical properties of the epidemic models. In this paper, based on some well known SIR epidemic models with time delays (see, for example, [1]-[4], [16]-[17], [20]), we propose the following delayed SIR epidemic model with density dependent birth rate,

$$\begin{cases} \dot{S}(t) = & -\beta S(t)I(t-h) - \mu_1 S(t) + b(1 - \beta_1 \frac{N(t)}{1+N(t)}), \\ \dot{I}(t) = & \beta S(t)I(t-h) - \mu_2 I(t) - \lambda I(t), \\ \dot{R}(t) = & \lambda I(t) - \mu_3 R(t), \end{cases} \tag{1.1}$$

where $S(t)+I(t)+R(t) \equiv N(t)$ denotes the number of a population at time t; $S(t)$, $I(t)$ and $R(t)$ denote the numbers of the population susceptible to the disease, of infective members and of members who have been removed from the possibility of infection through full immunity, respectively. It is assumed that all newborns are susceptible. The positive constants μ_1,μ_2 and μ_3 represent the death rates of susceptibles, infectives and recovered, respectively. It is assumed that $\mu_1 \leq \min\{\mu_2,\ \mu_3\}$. The positive constant b and λ

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