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REMARKS ON HYERS-ULAM STABILITY OF BUTLER-RASSIAS FUNCTIONAL EQUATION

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Abstract. It is known that the Butler-Rassias functional equation (1) is stable in the sense of Hyers and Ulam. In this paper, we will improve the previous result of the Hyers-Ulam stability of that equation.

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1 Introduction

In 1940, S. M. Ulam [11] gave a wide ranging talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of important unsolved problems. Among those was the following question concerning the stability of homomorphisms:

> Let G_1 be a group and let G_2 be a metric group with a metric $d(\cdot, \cdot)$. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$ then a homomorphism $H : G_1 \to G_2$ exists with $d(h(x), H(x)) < \varepsilon$ for all $x \in G_1$?

The case of approximately additive functions was solved by D. H. Hyers [5] under the assumption that G_1 and G_2 are Banach spaces.

Taking this fact into account, the additive Cauchy functional equation $f(x + y) = f(x) + f(y)$ is said to have the Hyers-Ulam stability. This terminology is also applied to the case of other functional equations. For more detailed definition of such terminology one can refer to [4, 6, 7].

In 2003, S. Butler [3] posed the following problem:

Problem 1 (Steven Butler) Show that for $c < -1$ there are exactly two solutions $f : \mathbf{R} \to \mathbf{R}$ of the functional equation, $f(x + y) = f(x)f(y) + c \sin x \sin y$.

Recently, Michael Th. Rassias excellently answered this problem by proving the following theorem (see [10]):