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STABILITY AND BIFURCATION ANALYSIS IN A NEURAL NETWORK MODEL WITH DELAYS

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Abstract. We consider a system of delayed differential equations representing a simple model for a ring of neurons in which each neuron receives two time delayed inputs: one from itself and another from the previous neuron. The stability and bifurcation are investigated by analyzing the distribution of the roots of the characteristic equation associated with the system. A bifurcation set is given in an appropriate parameter plane.
Keywords: Neural network; Delay; Stability; Bifurcation.
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1 Introduction

In 1984, Hopfield [11] considered a simplified neural network model in which each neuron is represented by a linear circuit consisting of a resistor and a capacitor and is connected to the other neurons via nonlinear sigmoidal activation functions. Assuming that the communication between neurons and the self-communication are instantaneous, Hopfield arrived at a ordinary differential system. In reality, neural networks often have time delays, for example due to the finite switching speed of amplifiers in electronic neural networks, or due to the finite signal propagation time in biological networks. In a first attempt to study this effect, Marcus and Westervelt [13] incorporated a single time delay into the connection term of Hopfield's model and observed sustained oscillations resulting from this time delay. Further study in Marcus and Westervelt's model can be found in [2,3,10,12,13,24,25] and the references therein.

In 1994, Baldi and Atiya [1] proposed a neural network which consists of a ring of neurons connected cyclically with delayed interactions. Different delays are introduced for the communication between the adjacent neurons. In recent years, there are many researchers who are interested in studying the stability of such neural network models with multiple delays by Liapunov's second method, we refer to [9,18,19,26]. Meanwhile, there are many researchers who has focused on the bifurcation analysis in two-neuron network model with two delays and tri-neuron network model with three delays. See for example [6,7,8,14,15,17,20-23].

In 1999, Campbell [4] generalized Baldi and Atiya's model [1] to a network that consists of a ring of neurons where the jth element receives two time