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FUNDAMENTAL THEORY OF CONTROL OF GENERAL FIRST-ORDER MATRIX DIFFERENCE SYSTEMS

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Abstract. This paper presents the general solution of the first-order matrix difference/discrete system

$$T(n+1) = A(n)T(n)B(n) + D(n)U(n)$$
$$R(n) = C(n)T(n)$$

in terms of two fundamental matrix solutions of $T(n+1) = A(n)T(n)$ and $T(n+1) = B^*(n)T(n)$. Then questions are addressed related to controllability, observability, and realizability. Further, more general criteria are presented for complete controllability and complete observability of time-invariant systems.
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1 Introduction

In this paper we shall be concerned with the general first-order matrix difference system

$$T(n+1) = A(n)T(n)B(n) + D(n)U(n) \tag{1.1}$$

$$R(n) = C(n)T(n)$$

where $A(n), B(n)$ and $T(n)$ are all square matrices of order s whose elements $a_{ij}(n)$, $b_{ij}(n)$, $d_{ij}(n)$ and $t_{ij}(n)$ are all real or complex functions defined on $N^+_{n_0}$ where

$$N^+_{n_0} = \{n_0, n_0 \pm 1, ...n_0 \pm k...\}$$

with $k \in N^+$ and $n_0 \in N, N$ being the set of integers. $U(n)$ is the control function and is of order $(r \times s)$; $D(n)$ is of order $(s \times r)$; the output signal $R(n)$ is $(p \times s)$, and $C(n)$ is also $(p \times s)$. Standard terminology is that the difference system (1.1) is said to be time invariant if the coefficient matrices $A(n)$ and $B(n)$ do not vary with time (i.e., is independent of n). Otherwise the system (1.1) is said to be time variant.

Controllability involves the effect of the input signal on the state matrix and does not involve the output equation. On the other hand, observability