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NONEXISTENCE OF POSITIVE SOLUTIONS FOR NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS

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Abstract. This paper studies nonexistence of positive solutions of a nonlinear neutral differential equation with an asymptotically periodic coefficient. Sufficient conditions for the nonexistence of positive solutions are obtained. Our results are based on the linearized methods and the establishment of an exponential estimate of positive solutions of certain linear inequality with periodic coefficient. Our approach utilizes the fact that the parameter set consisting of all points (p, τ, σ, q) such that the above inequality has no eventually positive solutions is open in certain metric space.
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1 Introduction

In this paper, we study nonexistence of positive solutions of the nth-order nonlinear neutral differential equation

$$[x(t) - P(t)g(x(t-\tau))]^{(n)} + Q(t)h(x(t-\sigma)) = 0, \tag{1.1}$$

under the following conditions:

(a) $n \geq 1$ is an odd integer,P,$Q \in C([t_0, \infty), R), g, h \in C(R, R), \tau > 0, \sigma \geq 0$.

(b) $\limsup_{t\to\infty} P(t) = p^* \in (0,1), \liminf_{t\to\infty} P(t) = p \in (0,1)$.

(c) $Q(t)$ is asymptotically periodic in the sense that there exist a continuous and positive τ-periodic function $q(t)$ and a function ε_t such that

$$Q(t) = q(t) + \varepsilon_t, \quad \varepsilon_t \to 0 \quad \text{as} \quad t \to \infty.$$

(d) $0 \leq \frac{g(u)}{u} \leq 1$ for $u \neq 0, \lim_{u\to 0} \frac{g(u)}{u} = 1$.

(e) $uh(u) > 0$ for $u \neq 0, \lim_{u\to 0} \frac{h(u)}{u} = 1$.

(f) $|h(u)| \geq h_0 > 0$ for $|u|$ sufficiently large.