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FUNDAMENTAL SOLUTION AND ASYMPTOTIC STABILITY OF LINEAR DELAY DIFFERENTIAL EQUATIONS

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Abstract. In this paper we formulate sufficient conditions for the asymptotic stability of linear delay systems of the form

$$\dot{x}_k(t) = -\sum_{\ell=0}^{m}\sum_{j=1}^{n} a_{kj}^{(\ell)} x_j(t-\tau_{kj}^{(\ell)}), \qquad k=1,\dots,n, \quad t\ge 0,$$

where $a_{kj}^{(0)}, a_{kj}^{(\ell)} \in \mathbb{R}$, $\tau_{kj}^{(0)}=0$, $\tau_{kj}^{(\ell)}\ge 0$, $k,j=1,\dots,n$, $\ell=1,\dots,m$. In order to apply our results, we give estimates for the integral $\int_0^\infty |v(t)|\,dt$, where v is the fundamental solution of certain associated scalar linear delay differential equations with multiple delays.

Keywords. linear delay differential equations, fundamental solution, asymptotic stability
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1 Introduction

Consider the delay system

$$\dot{x}_k(t) = -\sum_{j=1}^{n} a_{kj}x_j(t) - \sum_{j=1}^{n} b_{kj}x_j(t-\tau_{kj}), \qquad k=1,\dots,n, \quad t\ge 0, \tag{1.1}$$

where $a_{kj}, b_{kj}\in\mathbb{R}$, $\tau_{kj}\ge 0$, $k,j=1,\dots,n$. The stability of the trivial (zero) solution of special classes of (1.1) has been studied, e.g., [3]–[19]. In this paper we extend and improve these results for (1.1). Moreover, we formulate our results for the more general linear delay system

$$\dot{x}_k(t) = -\sum_{\ell=0}^{m}\sum_{j=1}^{n} a_{kj}^{(\ell)} x_j(t-\tau_{kj}^{(\ell)}), \qquad k=1,\dots,n, \quad t\ge 0, \tag{1.2}$$

where $a_{kj}^{(0)}, a_{kj}^{(\ell)} \in \mathbb{R}$, $\tau_{kj}^{(0)}=0$, $\tau_{kj}^{(\ell)}\ge 0$, $k,j=1,\dots,n$, $\ell=1,\dots,m$.

First we recall some known results for the stability of (1.1). All these results rely on the notion of an M-matix. A square matrix is called non-singular M-matrix, if all its off-diagonal elements are non-positive, and all its principal minors are positive. We refer, e.g., to [2] for many equivalent