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# QUADRATIC OPTIMAL CONTROL PROBLEM FOR DIFFERENTIAL SYSTEM OF PETROWSKY TYPE

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**Abstract.** In this paper, by using the Dubovitskii-Milyutin theorem we study the optimal control problem for $n \times n$ differential coupled system of Petrowsky type, so the system involving self-adjoint elliptic operator of $2\ell$ -th order, with an infinite number of variables. We drive the necessary and sufficient condition of optimality for a mixed Dirichelt problem with quadratic performance functional and constrained control. A characterization of the optimal control in terms of the adjoint system by a set of inequalities are given. Finally, the control problem with general performance functional is considered.

**AMS subject classification :** 35R15, 49J20, 49K20, 93C20.

## 1 Introduction

Interest in the analysis of function of infinitely many variables (understood here to be a vector in an infinite tensor product of one dimensional spaces) has increased considerably in recent years in connection with the problems and the advances in quantum field theory on the one hand, and with the purely mathematical desire to comprehend the situation in the analysis of point functions on infinite-dimensional spaces on the other hand [1].

The linear quadratic optimal control problem described by distributed parameter systems have a variety of mechanical and technical sources and applications. Fundamental class of optimal controls and its mathematical approaches can be found in Lions [13].

The optimality conditions for systems and $n \times n$ systems governed by different type of second order partial differential operator defined on spaces of functions of infinitely many variables are initiated and proved in [2-7, 10-12] such operators are similar to stationary Schrodinger operator. The interest in the study of this class of operators stimulated by problems in quantum field theory [1].

In [9], first we study a partial differential operator of $2\ell$ -th order with an infinite number of variables and then we find the set of inequalities defining