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OSCILLATION OF SOLUTIONS OF IMPULSIVE NEUTRAL DIFFERENCE EQUATIONS WITH CONTINUOUS VARIABLE

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Abstract. We obtain sufficient conditions for oscillation of all solutions of the neutral impulsive difference equation with continuous variable

$$\begin{cases} \Delta_\tau(y(t)+Py(t-m\tau))+Q(t)y(t-l\tau)=0, \quad t\geq t_0-\tau, \quad t\neq t_k, \\ y(t_k+\tau)-y(t_k)=b_ky(t_k), \quad k\in N(1), \end{cases}$$

where Δ_τ denotes the forward difference operator, i.e., $\Delta_\tau z(t)=z(t+\tau)-z(t)$, P is a constant, $Q(t)\in C([t_0-\tau,\infty),(0,\infty))$, m,l are positive integers, $\tau>0$ *and* b_k are constants, $0\leq t_0<t_1<t_2<\cdots<t_k<\cdots$ with $\lim\limits_{k\to\infty}t_k=\infty$.
Key words. Neutral difference equation with continuous variable; Impulse; Oscillation.
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1 Introduction

Let R denote the set of all real numbers. For any $a\in R$, define $N(a)=\{a,a+1,a+2,\cdots\}$. For any $t,\tau\in R$, $r\in N(1)$, define $N(t-r\tau,t-\tau)=\{t-r\tau,t-(r-1)\tau,\cdots,t-\tau\}$.

Consider the Neutral impulsive difference equation with continuous variable

$$\begin{cases} \Delta_\tau(y(t)+Py(t-m\tau))+Q(t)y(t-l\tau)=0, \quad t\geq t_0-\tau, \quad t\neq t_k, \\ y(t_k+\tau)-y(t_k)=b_ky(t_k), \quad k\in N(1), \end{cases} \tag{1}$$

where Δ_τ denotes the foreward difference operator, i.e., $\Delta_\tau z(t)=z(t+\tau)-z(t)$, P is constant, $Q(t)\in C([t_0-\tau,\infty),(0,\infty))$, m,l are positive integers, $\tau>0$ *and* b_k are constants, $0\leq t_0<t_1<t_2<\cdots<t_k<\cdots$ with $\lim\limits_{k\to\infty}t_k=\infty$. Set $l_0=max\{m,l\}$. For any $t_0\geq 0$, let $\phi_{t_0}=\{\varphi:[t_0-(l_0+1)\tau,t_0-\tau]\to R|\ \varphi(t)$ is piecewise continuous on $[t_0-(l_0+1)\tau,t_0-\tau]$, $\varphi(t)$ is finite for every $t\in[t_0-(l_0+1)\tau,t_0-\tau]$, the right and left limit $\varphi(t^+)$ and $\varphi(t^-)$ of $\varphi(t)$ exist for every $t\in(t_0-(l_0+1)\tau,t_0-\tau)$, and $\varphi((t_0-(l_0+1)\tau)^+)$ and $\varphi((t_0-\tau)^-)$ exist $\}$.