Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 47-75

LIPSCHITZIAN ETIMATES IN DISCRETE-TIME CONSTRAINED OPTIMAL CONTROL

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Abstract. This paper is devoted to the analysis of a finite horizon discrete-time stochastic optimal control problem, in presence of constraints. We study the regularity of the value function which comes from the dynamic programming algorithm. In particular, we derive estimates of the Lipschitz constant of the value function, by means of a regularity result of the multifunction that defines the admissible control set.
Keywords. Optimal Control, Dynamic Programming, Constraints, Lipschitz regularity, Multifunctions.
AMS (MOS) subject classification: 49L20, 49N60, 32A12, 37N40, 37N35.

1 Introduction

This paper is concerned with the analysis of a general Finite Horizon Discrete-Time Stochastic Optimal Control Problem, in presence of constraints. We focuse our attention on the maximization of a terminal cost function (*utility function*).
The aim of this work is to give a method to establish the Lipschitz regularity of the value functions which arise when the classical dynamic programming algorithm [4] is applied to solve the related optimization problem. The main source of difficulty is the presence of a set of controls which depends on the state of the system. Actually, the regularity of the value function, in stochastic optimization problems with constraints on the admissible controls, depends on the regularity of the *marginal function* [2] and, the regularity of the marginal function is connected to the regularity of the multifunction which defines the set of the admissible controls, see also [5].
The relevance of these results is related to optimization problems, nonlinear analysis, mathematical economics, etc. Lipschitz type properties are important to study mappings associated with the solutions of optimization problems, including variational inequalities, and mathematical programs [16].
The motivation for which a bound on the Lipschitz constant is useful can be explained by two main objectives: for either (i) convergence results of numerical schemes or (ii) a-priori estimates. In proving convergence of numerical