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EXISTENCE OF SOLUTIONS FOR A CLASS OF IMPLICIT DIFFERENTIAL EQUATIONS IN BANACH SPACES[1]

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Abstract. In this paper, by using the directional Lipschitzian condition and Krasnoselskii's fixed point theorem, we prove some new existence theorems of solution for a class of first-order implicit ordinary differential equations in Banach spaces.
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1 Introduction

In this paper, we study the existence of solutions $x : J \to E$ of the Cauchy problem consisting of the first-order implicit ordinary differential equation

$$\begin{cases} x'(t) = f(t, x(t), x'(t)) + g(t, x(t)), \\ x(t_0) = x_0 \in E, \end{cases} \tag{1.1}$$

where E is a Banach space, $R = (-\infty, +\infty)$, $t_0 \in R$, $x_0, u_0 \in E$, $a, b, c \in R^+ = (0, +\infty)$, $J = [t_0, t_0 + a]$, $D = J \times B_b(x_0) \times B_c(u_0)$, $W = J \times B_b(x_0)$, $f : D \to E, g : W \to E$, and $B_r(\omega)$ denotes the ball which center at ω and radius is r.

The study of such types of problems is motivated by an increasing interest in the differential equations with applications in Banach spaces and implicit differential equations under various initial and boundary conditions (see, for example, [3]-[10], [12], [14] and the references therein). In 1978, Benavides [1] studied the implicit differential equations in Banach spaces and presented some existence theorems of solution for the implicit differential equations in Banach spaces. Recently, Wang and Cui [15] proved an existence theorem of a unique solution for the initial value problem of the first order implicit ordinary differential equation: $x'(t) = f(t, x(t), x'(t))$ under the condition

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