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EXPONENTIAL DICHOTOMY AND ADMISSIBILITY FOR EVOLUTION FAMILIES ON THE REAL LINE

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Abstract. We give necessary and sufficient conditions for uniform exponential dichotomy of discrete evolution families in terms of the admissibility of the pairs $(l^\infty(\mathbf{Z},X), c_0(\mathbf{Z},X))$, $(l^\infty(\mathbf{Z},X), l^\infty(\mathbf{Z},X))$ and $(c_0(\mathbf{Z},X), c_0(\mathbf{Z},X))$, respectively. We prove that the uniform exponential dichotomy of an evolution family is equivalent with the uniform exponential dichotomy of the discrete evolution family associated to it. Thus, we obtain that the uniform exponential dichotomy of an evolution family is equivalent with the admissibility of one of the pairs $(l^\infty(\mathbf{Z},X), c_0(\mathbf{Z},X))$, $(l^\infty(\mathbf{Z},X), l^\infty(\mathbf{Z},X))$ or $(c_0(\mathbf{Z},X), c_0(\mathbf{Z},X))$, and the uniform exponential dichotomy of a strongly continuous evolution family is equivalent with the admissibility of one of the pairs $(C_b(\mathbf{R},X), C_0(\mathbf{R},X))$, $(C_b(\mathbf{R},X), C_b(\mathbf{R},X))$ or $(C_0(\mathbf{R},X), C_0(\mathbf{R},X))$, respectively. Finally, we apply our results at the characterization of the exponential dichotomy of C_0-semigroups.

Keywords. Evolution family, discrete evolution family, uniform exponential dichotomy, admissibility, C_0-semigroup.
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1 Introduction

Exponential dichotomy is one of the most important asymptotic properties of evolution equations. In the last decades, dichotomy became a classical and a well studied subject (see [1]-[9], [13]-[20], [22], [24], [25], [28]-[30]). Starting with the pioneering work of Perron, the asymptotic properties of the solutions of the equation $\dot{x} = A(t)x$ on a Banach space X, have been expressed in terms of the specific properties of the operator $Px(t) = \dot{x}(t) - A(t)x(t)$ on a space of X-valuated functions (see [8], [9], [17], [27]). More recent, for the case of evolution families $\mathcal{U} = \{U(t,s)\}_{t,s\ \in J,\ t\geq s}$, instead of the operator P, one started to investigate the integral equation

$$(E_{\mathcal{U}}) \qquad \varphi(t) = U(t,s)\varphi(s) + \int_s^t U(t,\tau)v(\tau)\ d\tau, \quad t \geq s, t, s \in J,$$

where $J \in \{\mathbf{R}_+, \mathbf{R}\}$. An important result for the case $J = \mathbf{R}_+$, has been proved by Van Minh, Räbiger and Schnaubelt in [25] and it is given by