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Hyperbolic Real Quadratic Cellular Automata

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Abstract. In this paper we develop some techniques to obtain global hyperbolicity for a certain class of endomorphisms of $\mathbb{R}^n$ called real cellular automata, which are characterized by the property of commuting with a shift. In particular, we show that one parameter families of generic quadratic cellular automata in $\mathbb{R}^n$ are hyperbolic for large values of the parameter.

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1 Definitions and statement of results

Several mathematical and computational models, as those that arise from biological networks, image processing or fluid dynamics by discretizing ordinary differential equations and from the qualitative analysis of the evolution of spatially extended dynamical systems given by partial differential equations, take us to the study of a special class of dynamical systems known as Lattices Dynamical Systems (LDS). Roughly speaking, a LDS is an infinite system of ordinary differential equations (continuous time) or difference equations (discrete time).

In order to define a discrete time lattice dynamical system, let Ω be a lattice (with discrete structure) whose elements are called cells (or sites). For each $\omega \in \Omega$, let X_ω be a topological space (in most applications those spaces are the same) and $\mathcal{M} = \prod_{\omega\in\Omega} X_\omega$ endowed with the product topology. A LDS is a pair $(\mathcal{M}, F)$, where $F = \{F_\omega\}_{\omega\in\Omega} : \mathcal{M} \to \mathcal{M}$ is a product structure preserving mapping, also called the global transition function, that is $F(\{x_\omega\}_{\omega\in\Omega}) = \{F_\omega(x)\}_{\omega\in\Omega}$. The state-transition in $(\mathcal{M}, F)$ is given by the difference equation $x(n+1) = F(x(n))$, where $x(n) = \{x_\omega(n)\}_{\omega\in\Omega} \in \mathcal{M}$ for every $n \in \mathbb{Z}_+$. Cellular Automata (CA) are LDS's for which $\Omega = \mathbb{Z}^k$