*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **A:** *Mathematical Analysis* **12 (2005)** 769-781


# INTERVAL CRITERIA FOR OSCILLATION OF CERTAIN SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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**Abstract.** By employing a generalized Riccati technique and an integral averaging technique, new interval oscillation criteria are established for second-order nonlinear differential equations of the form $(r(t)y'(t))' + Q(t, y(t), y'(t)) = 0$.

**Keywords.** Nonlinear differential equations; Oscillation; Interval criteria; Generalized Riccati technique; Integral averaging technique.

**AMS (MOS) subject classification:** 34C10

## 1 Introduction

Consider the second order nonlinear differential equation

$$(r(t)y'(t))' + Q(t, y(t), y'(t)) = 0 \tag{1.1}$$

on the half-line $[t_0, \infty)$. In Eq. (1.1), we shall assume that the following conditions are satisfied:

(A1) the function $1/r \in L_{loc}([t_0, \infty), \mathbb{R})$, the set of real-valued, locally integrable functions on $[t_0, \infty)$, and $r > 0$ *a.e.* on $[t_0, \infty)$;

(A2) the function $Q(t, y, z) : [t_0, \infty) \times \mathbb{R}^2 \to \mathbb{R}$ is locally integrable for $t$ on $[t_0, \infty)$ and continuous for $y$ and $z$.

We recall that a function $y : [t_0, t_1) \to \mathbb{R}, t_1 > t_0$ is called a solution of Eq. (1.1) if $y(t)$ satisfies Eq. (1.1) for all $t \in [t_0, t_1)$. In the sequel, it will be always assumed that solutions of Eq. (1.1) exist for any $t_0 \geq 0$. A solution of Eq. (1.1) is called oscillatory if it has arbitrarily large zeros, otherwise it is called nonoscillatory. Finally, Eq. (1.1) is called oscillatory if all its solutions are oscillatory.

The oscillation problem for various particular cases of Eq. (1.1) such as the second order linear differential equation

$$(r(t)y'(t))' + q(t)y(t) = 0, \tag{1.2}$$

the second order nonlinear differential equations

$$(r(t)y'(t))' + p(t)y'(t) + q(t)f(y(t)) = 0, \tag{1.3}$$