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THE RELATIONSHIP BETWEEN DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT AND THE ASSOCIATED DISCRETE EQUATIONS, VIA DICHOTOMIC MAPS

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Abstract. Dichotomic maps are considered by means of the stability and asymptotic stability of the null solution of a class of differential equations with argument $[t]$ via associated discrete equations, where [.] designates the greatest integer function.

Keywords. Piecewise constant argument, stability, dichotomic maps

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1 Introduction

The study of differential equations with piecewise continuous argument has been the subject of many recent investigations such as Aftabizadeh & Wiener [1], Carvalho & Cooke [3], Carvalho & Marconato [4], Cooke & Wiener [5,6,7], Marconato [8,9] and Marconato & Spezamiglio [10]. Some equations of this type are similar in structure to those found in certain "sequential-continuous" models of disease dynamics as treated by Busenberg & Cooke [2].

Those equations include, as a particular case, the differential equations with argument constant in intervals, such:

$$x'(t) = f(t, x(t), x([t])) \tag{1.1}$$

where $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous map with $f(t, 0, 0) = 0$ for all $t \in \mathbb{R}$. A potential application of this equation is in the stabilization of hybrid control system, by which we mean one with a continuous plant and with a discrete controller.

We further impose that f takes bounded sets into bounded sets and satisfies enough additional smoothness conditions to ensure the existence, uniqueness and continuous dependence with respect the initial conditions, of the solutions of (1.1). The retarded of the equation (1.1) is sectionally continuous because $r(t) = t - [t]$ is a continuous map in $[n, n + 1)$, for all $n \in \mathbb{Z}$, and discontinuous at integers values. Our purpose is to study the stability of such equations by means of the analysis of the stability of the associated discrete equation, via dichotomic maps. It is possible to obtain recurrence