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Global Weak Solutions for a Periodic Integrable Shallow Water Equation with Linear and Nonlinear Dispersion

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Abstract. We prove the existence and uniqueness of global weak solutions for a periodic integrable shallow water equation with linear and nonlinear dispersion describing the unidirectional propagation of spatially periodic surface waves on a shallow layer of water.
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1 Introduction

Recently, Dullin, Gottwald, and Holm [15] derived a new equation describing the unidirectional propagation of spatially periodic surface waves on a shallow layer of water

$$\begin{cases} u_t - \alpha^2 u_{txx} + c_0 u_x + 3uu_x + \gamma u_{xxx} = & \alpha^2(2u_x u_{xx} + uu_{xxx}), \\ & t > 0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \\ u(t,x+1) = u(t,x), & t \geq 0, \quad x \in \mathbb{R}. \end{cases} \tag{1.1}$$

Here the constants α^2 and $\frac{\gamma}{c_0}$ are squares of length scales, and c_0 is a nonnegative parameter related to the linear wave speed in shallow water, and $u(t,x)$ stands for the fluid velocity. The equation, which is derived by the method of asymptotic analysis and a near-identity normal form transformation from water wave theory, combines the linear dispersion of the KdV equation with the nonlinear/nonlocal dispersion of the Camassa-Holm equation. It is completely integrable [15].

With $\alpha = 0$ in Eq.(1.1) we find the well-known Korteweg-de Vries equation which describes the unidirectional propagation of waves at the free surface of shallow water under the influence of gravity. $u(t,x)$ represents the wave height above a flat bottom, x is proportional to distance in the direction of propagation and t is proportional to elapsed time. The Cauchy problem of the KdV equation has been studied extensively and, as soon as $u_0 \in H^1(\mathbb{R})$, the solution of the KdV equation is global (see [18]). The equation is completely integrable (see [19]).